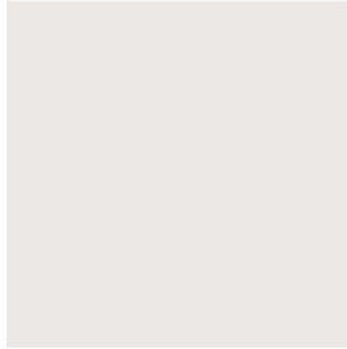
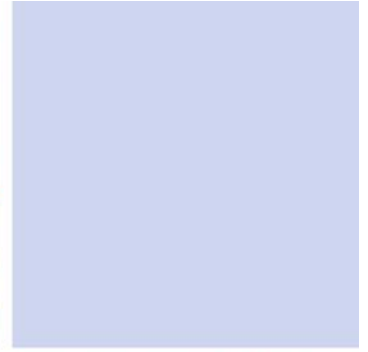




Freddie Mac Working Paper



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Buchi Ramagopal is Director of Financial Research at Freddie Mac, McLean, VA; Douglas McManus is a Principal Economist at Freddie Mac, McLean, VA; and Yan Chang is an Economist, Freddie Mac, McLean, VA.

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Your comments and questions are welcome. Address correspondence to Buchi Ramagopal, 8200 Jones Branch Drive MS 484, McLean, VA 22102; (703) 903-4211; buchi_ramagopal@freddiemac.com. Media Contact: Eileen Fitzpatrick (703) 903-2446; eileen_fitzpatrick@freddiemac.com.

Abstract

This paper examines the impact of mortgage hedging activities on interest rate volatility. A recent analysis by Perli and Sack [2003] provides a framework for evaluating this question along with estimates of the extent to which hedging by agents in the mortgage market amplifies interest rate volatility. We evaluate the amplification issue by first considering a wider range of interest rate derivatives to isolate the sources that might be contributing to volatility. Second, by considering a rolling window over which we obtain estimates of key parameters we show that the measured effects of mortgage hedging on volatility are distorted by two outlier episodes: the collapse of LTCM and the market reaction in the aftermath of the 9/11 tragedy. Eliminating these episodes from the data shows that hedging appears to stabilize rate volatility over some periods while exacerbating it in others. We conclude that no simple relationship between hedging and rate volatility is implied.

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Introduction

Volatility is an ever-present feature of financial markets. Recent history has evidenced several episodes of heightened volatility. Events such as the Russian debt crisis, the collapse of LTCM, and the disruption of the Asian markets in the late 1990s have all been responsible for periods of heightened volatility. While such volatility can be viewed from a macroeconomic perspective as the source of economic instability, from a complete markets perspective, this volatility represents the efficient functioning of markets in the face of uncertainty. Over the past two decades, markets, institutions, and practices have been examined as possible sources of 'excessive' volatility. This paper examines the possible role of mortgage hedging in creating volatility in interest rate markets.

Starting in 1990, 30-year fixed mortgage rates declined from a high of 10.5% to a low of 5.2% in June of 2003. This decline in rates led households to refinance their mortgages in record numbers. The refinancing activity peaked during the years of 1992-1993, 1998 and most recently, in 2003. By the end of 2003, up to 40-45% of the \$7.0 trillion single-family debt outstanding was financed in 2003 alone. The volume of refinancing activity in 2003, for example, led to close to \$3 trillion of mortgage debt outstanding to be re-struck at lower rates. Despite the huge volume, the US mortgage market managed to respond flexibly, providing homeowners the benefit of lower rates.

The decline in rates and the re-striking of mortgage debt led an increasing number of mortgage market participants such as originators, mortgage-servicers, and portfolio managers to manage the risks associated with this secular decline in rates. Fixed rate mortgages have positive duration and negative convexity, that is, as market interest rates rise, then the value of the security would decline linearly with duration and quadratically with convexity. As such, market participants can choose a mixture of static and dynamic hedging strategies to mitigate the associated interest rate risk. Static hedging of mortgage interest rate risk can be accomplished by issuing callable debt or through the synthesis of callable debt engineered by the purchase of swaptions along with the issuance of bullet debt. Dynamic hedging, on the other hand, calls for the constant management of the relative durations of the assets and liabilities, typically through

swap transactions. Counterparties to these derivative transactions have diversified books of business and can consequently manage these risks. However, they also serve as intermediaries in the market for these transactions by identifying other parties that might naturally offset these mortgage market risks.

The nature and magnitude of mortgage hedging activities has raised the issue of its impact on market interest rates. Kambhu and Mosser [2001] make the point that “disruptions to liquidity can affect the short-term dynamics of interest rates and the shape of the curve independently of fundamentals.” Their analysis shows that the dynamics of the yield curve have changed in ways that seem related to the growth of the interest rate options market. Several approaches have been taken to evaluating the impact of large mortgage market participants on rate volatility. Naranjo and Toevs [2002] and Gonzalez-Rivera [2001] suggest that large mortgage market participants reduce volatility in mortgage rate spreads as a result of their participation in the market. Alternatively, characterizations by Perli and Sack [2003] and Goodman and Ho [2004] suggests that mortgage hedging raises long-term rate volatility, though their estimates of the extent of amplification of rates resulting from hedging differ. The argument of Perli and Sack suggests that the tools of financial engineering, as applied to mortgage risk management, might be responsible for heightened volatility. This paper investigates this claim.

The issue of whether rational speculators or risk management frameworks can destabilize prices or exacerbate volatility has been a subject of continued research. One position, starting with Friedman [1953], is that rational speculators act to stabilize asset prices. In Friedman’s view “Speculators who destabilized prices by, on average, buying when prices were high and selling when they were low would find this unsustainable and be eliminated from the market. The more rational speculators who moved prices back toward fundamentals would eliminate such speculators, who are less than rational and moved prices away from fundamentals.” Alternatively, several recent researchers have developed theoretical examples in which trading activities can destabilize prices. In these examples, rational speculators might not easily stabilize asset prices even when they have access to identical information, and have identical priors as shown by Hart and Kreps [1986]. Similarly, DeLong, Schleifer, Summers and Waldmann [1990] also show, for example, that investors with extrapolative expectations, those who chase trends, or have stop-loss orders in place, all of which could be characterized as positive feedback investment strategies, might contribute to destabilizing market movements.

Arguments similar to those of destabilizing speculation have been used to make the case that risk management in general, or portfolio insurance in particular, contributes to greater volatility in rates or asset prices. Portfolio insurance, for example, according to the Brady Commission [1988], was considered to have been the exacerbating factor in the crash of 1987.

In a similar vein, the Value-at Risk (VaR) methodology has also been blamed for amplifying volatility (See Persaud [2000] and Dunbar [2000]). The argument is that shocks might raise volatility initially, leading to a breach of VaR positions and require investors to raise the amount of allocated capital; otherwise investors would be required to liquidate some part of their portfolios. It further raises the possibility that large institutions that exceed predetermined VaR measures (determined internally by management or, externally by a regulator) might then simultaneously “sell the same asset at the same time, creating higher volatility and correlations, which exacerbate the initial effect, forcing additional sales.”¹ However, Jorion [2002] claims that such criticisms relating to the VaR framework have been misplaced.

In effect, the argument presented by Perli and Sack is that when interest rates rise, mortgage hedgers raise the duration of their liabilities by executing a pay-fixed swap, i.e., raising the duration of their liabilities by synthetically exchanging short term debt for longer maturity obligations. Conversely, in response to declining rates mortgage hedgers will reduce the duration of their liabilities by conducting receive-fixed swaps. The magnitude of the amplification effects will depend on the degree to which mortgage hedgers adopt dynamic rather than static hedging strategies. Underlying their empirical method is the assumption that some constant proportion of mortgage interest rate risk is actively managed through dynamic rather than static hedging strategies.

While the arguments presented by Perli and Sack are appealing, their “amplification proposition” i.e., mortgage hedging amplifies rate movements, is similar to arguments that have been made in different settings. A recent line of argument has been to push for the increased use of adjustable rate mortgages, which would reduce the need to dynamically hedge. While the adoption of adjustable rate mortgages would eliminate the need on the part of large mortgage holders to dynamically hedge, it ignores the possible impact on individual homeowners and the fact that they may not be in a position to efficiently manage such interest rate risk as evidenced by higher ARM default rates for otherwise identical mortgages.

¹ Jorion [2002], p. 2.

It is in fact ironic that ARMs are being promoted as an alternative to fixed rate mortgages in the US market at a point when countries such as the United Kingdom see the prevalence of ARMs as being one of the reasons for their boom-bust housing cycles, which increases the amplitude of their business cycles. The review by David Miles [2004] highlights the need for the UK housing market to introduce longer term fixed-rate mortgages with prepayment options, much like those in the US mortgage market.

Still, the question remains as to whether or not dynamic hedging by mortgage market participants creates volatility, and if so, the extent to which this is the case. Ultimately, the impact of mortgage hedging on interest rate volatility is one that has to be determined empirically.

In the interest of consistency, we use a framework identical to that of Perli and Sack and lay out the framework in section 2. The data along with results of the first stage regression are described in section 3. The results of the first part of the empirical exercise, measuring the extent to which mortgage hedging exacerbates volatility is presented in section 4. The final arguments and summary of results are presented in the conclusion.

The Model

The underlying intuition of the framework utilized is one where changes in long-term rates are driven by a data generating process. The shocks generated by this process emanate from monetary policy innovations, news and changes in risk preferences. The long-term rate in question is the ten-year swap rate and changes in this rate between time t and $t+n$ are generated by a “fundamental” shock $\varepsilon(t, t+n)$. The changes in the ten-year swap rate are characterized as:

$$\Delta r(t, t+n) = \sqrt{\gamma_t} * \varepsilon(t, t+n) \quad (1)$$

In this setting, the fundamental shock $\varepsilon(t, t+n)$ is exacerbated by a factor $\sqrt{\gamma_t}$, whose value is known at time “ t ” and whose magnitude is determined by the degree of mortgage hedging activity. This amplification factor is supposed to characterize the state of the mortgage market at the beginning of the period. It takes on a value that could be close to one if the extent of prepayment risk is low, and values that are higher if prepayment risk is greater. Thus, shocks during periods of high prepayment risk are amplified, and shocks during periods of low prepayment risk are dampened.

Using the specification in (1), the variance of the interest rate is characterized as:

$$\sigma_{\Delta r, t}^2 = \gamma_t \sigma_{\varepsilon, t}^2 \quad (2)$$

The expression describes the second moment of the ten-year swap rate as a function of the second moment of the fundamental shock and the amplification factor. The conditional variance of the fundamental shocks is described as following an AR(1) process:

$$\sigma_{\varepsilon,t}^2 = \alpha_0 + \alpha_1 \sigma_{\varepsilon,t-1}^2 + u_t \quad (3)$$

Using equation (2) to rewrite last period's fundamental variance as a ratio of last period's variance of the swap rate to the amplification factor and using (3), equation (2) is expressed as:

$$\sigma_{\Delta r,t}^2 = \alpha_0 \gamma_t + \alpha_1 \frac{\gamma_t}{\gamma_{t-1}} \sigma_{\Delta r,t-1}^2 + \gamma_t u_t \quad (4)$$

During periods of higher prepayment risk when γ is higher it raises the average level of volatility, leading to a larger value of the first term on the right hand side of (4). The second term above highlights the autocorrelation of volatility, and third term describes the “volatility of volatility” effect. This term describes the manner in which changes in expected volatility are greater when prepayment risk is high.

The framework for determining the magnitude of the amplification factor $\sqrt{\gamma_t}$, is described as:

$$\gamma_t = 1 + \beta X_t \quad (5)$$

where X_t is a measure of hedging intensity. When there is no hedging pressure the value of γ_t is one, and higher when there is an increased need for greater mortgage market hedging. Various proxies of mortgage market hedging needs are considered and we discuss them further in the next section.

Using the specification for the hedging amplification factor in (5), (4) can be rewritten as

$$\sigma_{\Delta r,t}^2 = \alpha_0 + \alpha_0 \beta X_t + \alpha_1 \frac{1 + \beta X_t}{1 + \beta X_{t-1}} \sigma_{\Delta r,t-1}^2 + (1 + \beta X_t) u_t \quad (6)$$

The final specification uses a lagged value of the hedging amplification factor to avoid the endogeneity problem that arises from the possibility that volatility both causes and is caused by hedging. This gives rise to the following specification:

$$\sigma_{\Delta r,t}^2 = \alpha_0 + \alpha_0 \beta X_{t-1} + \alpha_1 \sigma_{\Delta r,t-1}^2 + (1 + \beta X_{t-1}) u_t \quad (7)$$

The product of the parameters $\alpha_0 \beta$ provide a measure of the degree of sensitivity of volatility to changes in hedging intensity. Note that the error term is multiplied by a factor relating to hedging intensity that induces heteroscedasticity into the estimated equation, which is

interpreted as a “volatility of volatility” effect. In its current form, (7) is estimated using the maximum likelihood framework, where the likelihood function is given by:

$$L(\alpha_0, \alpha_1, \beta, \nu) = -\frac{1}{2} \ln(\nu(1 + \beta X_{t-1})^2) - \frac{1}{2\nu} (\sigma_{\Delta r, t}^2 - \alpha_0(1 + \beta X_{t-1}) - \alpha_1 \sigma_{\Delta r, t-1}^2)^2 \quad (8)$$

The Data

Volatility in financial markets is typically measured in one of two ways. The first examines the ex-post historical volatility of rates. The second uses the financial derivative pricing models to ascertain the ex-ante volatilities consistent with derivative asset prices. This paper, like Perli and Sack, will focus on the volatilities implied by the prices of swaption derivative contracts.² These implied volatilities are the result of applying Black’s (1976) model of interest rate option pricing to translate market data on prices to volatility.

The Perli and Sack methodology seeks to explain short-term movements in volatilities unrelated to longer-term shifts in the underlying interest rate process. To implement this measure they derive the residual component of short-term volatility orthogonal to long-term volatility. This step involves regressing a squared measure of short-term implied volatility on squared values of longer-term implied volatility, and using the residual as the dependent variable in the analysis.

The residual η_t in (9) below measures the part of short-term volatility not explained by long-term volatility:

$$\sigma_{\Delta r, t}^2(3m \times 10y) = \psi_0 + \psi_1 \sigma_{\Delta r, t}^2(2y \times 10y) + \eta_t \quad (9)$$

where $\sigma_{\Delta r, t}^2(3m \times 10y)$ is the square of the implied volatility of a three-month swaption on a ten-year swap. Similarly, $\sigma_{\Delta r, t}^2(2y \times 10y)$ is the square of the implied volatility on a two-year option on a ten-year swap. The results from (9) are used to build the orthogonalized measure of variance, which is constructed as follows:

$$\sigma_{\Delta r, t}^2 = \eta_t + \bar{\sigma}_{\Delta r, t}^2(3m \times 10y) \quad (10)$$

where η_t is the time series of residuals and $\bar{\sigma}_{\Delta r, t}^2(3m \times 10y)$ is the sample average of the variance of the 3 month x 10 year swaptions.

² Swaptions are options contracts on a swap. These contracts provide the holder with an option to enter into a swap at some point in the future. As is the practice in these markets the quoted prices of these contracts are in volatility units, the market determined value of ex-ante volatility as implied by Black’s model.

We use time series on the implied volatility for four swaptions: 3 month x 5 year, 3 month x 10 year, 6 month x 5 year and 6 month x 10 year. The implied volatilities are measured in basis points (bps.), i.e. 100th of one percent. The sources for the option volatilities were Salomon Smith Barney's *Yield Book* and Perli and Sack. The time series for each of these volatilities was constructed as the product of daily rate volatility and the appropriate forward rate.³ The descriptive statistics for the variables used and their definitions are described below in Exhibit 1.

The first variable described in Exhibit 1, mortgage market convexity measures the convexity exposure in terms of the magnitude of swaps that would have to be traded to remain duration neutral in the presence of a rate shock of 50 bps. We use the weekly observations on the mortgage market convexity measure, obtained from Perli and Sack to replicate their results. The remaining eight weekly series measure implied volatility of both short and long-term swaptions.

Exhibit 2 displays both short and long volatility series. In examining the data we noticed that the Perli and Sack "long" implied volatility (2yx10y) series deviated from our series starting in 2001. While the Perli and Sack series differed in these later months they generally agreed with our pre-2001 data. To check the reliability of these series we obtained additional independent volatility data from JP Morgan and LehmanLive.com and found that their data more closely corresponded to our series than the Perli and Sack series over the 2001 to 2003 period. As a result of this divergence we will consider model estimates using both the Perli and Sack series and the series using data from Salomon Smith Barney.

First we replicate the first-stage results of Perli and Sack using their series and these findings are given in the first row of Exhibit 3. Perli and Sack use the residuals from this first stage regression (after adding the average squared volatility) as the dependent variable in their second stage regression. Note that we numerically replicate their coefficient estimates.

The first set of results highlighted in the first row of Exhibit 3 below are obtained using Perli and Sack data, whereas, all the other results are obtained using our implied volatility series. It is noticeable that using our series the value of the coefficients and the R^2 are all significantly lower than those obtained by Perli and Sack.⁴ We then re-estimate this first stage using the

³ Basis point volatility for the 3 month x 10 year swaption equals the product of the rate volatility for the 3 month x 10 year and the 3 month forward ten year rates.

⁴ We also note that after replicating the Perli and Sack first-stage results, the R^2 of 0.60 differed from the 0.95 value reported in the Perli and Sack paper. It is important to note that this residual volatility is small both in relative and absolute magnitude. The average squared volatility is 1.07, of which the unexplained component is 0.21, with the

alternative data series for both short and long volatility. These results are given in the second row of Exhibit 3 and qualitatively replicate the signs of the Perli and Sack coefficient estimates, but differ materially in their magnitude.

In order to evaluate the sensitivity of the Perli and Sack findings, we perform alternative first stage regressions using different swaption structures. The third row varies the Perli and Sack experiment by changing the short volatility series to one based on 6m x 10y swaptions. The fourth row varies the Perli and Sack experiment by shortening the long volatility series to the 2y x 5y swaption. The fifth and final row alters both series, taking the 6m x 5y swaption volatility as the short series and the 2y x 5y series as the long series. In these three cases the sign and the magnitude of the coefficients are similar to what was obtained in the replication of the Perli and Sack model using our short and long series. While there may be a difference between the results based on the data sources, we find that the first stage regressions produce similar results across the range of swaption volatilities that we explore. This suggests that these Perli and Sack findings are not specific to the particular swaption contracts they chose to explore.

It is informative to examine the graph of the dependent variable created out of the first stage regression. Exhibit 4 plots the volatility series created by adding the residual from the first stage regression to the average squared volatility for the short-term swaption. Two features are striking in this graph. They are the spikes that occur the latter half of 1998 and near the end of 2001, associated with the Long Term Capital Management (LTCM) and the 9/11 events. The LTCM extremum is approximately a 6.54 standard deviation event and the 9/11 instance is a 3.77 standard deviation event. Since both least squares and Maximum Likelihood methodologies can be highly sensitive to outlier observations, this graph motivates the exploration of the sensitivity of the Perli and Sack findings to these stressful episodes.

With the possibility of model error and parameter instability, the issue remains as to whether any variability remains to be explained. However, our regressions (with R^2 ranging from .78 to .91) indicate that 9% to 22% of the variability remains to be explained. This is an issue that we will pursue in the next section.

The exogenous variable used in this analysis is a proxy for hedging activity.⁵ The interest rate risk in the universe of mortgage debt is measured by the total mortgage market duration and

coefficient of variation taking a value of 0.19, a small magnitude. In fact, it might be argued that this effect (of residual volatility) is so small that it is not of any great consequence to any policy issues that relate to stability in financial markets.

⁵ Perli and Sack consider three proxies for hedging. Since they indicate that convexity is the most robust and indicative measure of hedging intensity, we restrict our attention to this proxy.

convexity. Starting from a duration neutral position for a given portfolio, the measure of hedging intensity will depend on the changes in the level of duration of mortgage debt. This change in duration will depend, in turn, on the degree of convexity, as convexity is a measure of the rate of change of duration. As such, convexity serves as a reasonable proxy for changes in duration for investors who choose to dynamically hedge changes in duration.

It is worth noting that dynamic hedging is a reactive hedging protocol. However, most firms use a mixture of static and dynamic hedging. When rate changes alter the duration of assets, the portfolio can be rebalanced to a duration neutral position by changing the duration of liabilities, for example, by the use of swaps. To the extent that firms anticipate hedging needs and eliminate some or all of the convexity exposure through the use of swaptions to offset the negative convexity inherent in the mortgage universe, the residual negative convexity would be the appropriate measure of hedging intensity. However, residual post-hedging convexity is unobservable and so the analysis uses mortgage market convexity. If firms change their mix of dynamic and static hedging during the sample period, this could diminish the predictiveness of this regressor.

The series for convexity used in this paper is the same as that used by Perli and Sack. It provides a weekly measure of ten-year swap equivalents needed to swap out the changes in duration resulting from changes in rates during the same period.⁶

Impact of Mortgage Hedging on Volatility

Perli and Sack measure the impact of mortgage hedging on volatility by regressing the constructed residual short-term volatility variable on a proxy for mortgage market hedging, the total mortgage market convexity. First, we replicate the Perli and Sack second-stage analysis and obtain coefficients that are identical and are presented in the first shaded row of Exhibit 5. We then extend the second-stage analysis to different swaption structures. Using three other swaption volatilities for the same sample period (6mx10y, 3mx5y, and 6mx5y) we find that we are able to obtain the estimates of the coefficients α_0 and α_1 that are nearly identical and significant. For the parameter of interest, $\alpha_0\beta$, the sign is in agreement with the results of Perli and Sack for all of the swaption structures, but the magnitude is substantially reduced. We find that the estimates do not appear significant, but the reliability of inferences drawn is questionable

⁶ Ten-year swap equivalents are a standard measure of the number of dollars of this contract that must be used to eliminate a duration exposure.

because the standard errors are not adjusted for heteroscedasticity and the confidence intervals do not acknowledge the possible cointegration of these volatility series.

We move next to the estimation of the parameters using MLE with the specification described in (8). The MLE is efficient and provides a basis for asymptotically valid inference. The results for this exercise are described in Exhibit 6 below. With the MLE estimation exercise we are estimating four parameters: α_0 , α_1 , β and ν , and the parameter of interest is β , the amplification parameter. For all of the swaption structures for this sample period, we find that the coefficients are all highly significant and closely approximate the results of Perli and Sack.⁷

One important test of a models' validity is parameter stability. Intuitively, there maybe structural differences between periods with low levels of hedging activity as compared with periods of high levels of hedging activity. For example, in periods where the demand for hedging is low, risk managers may increase the proportion of hedging accomplished through static hedges. We proceed to examine this issue by segmenting the sample period into a period when the MBA Refi-index was low and with expected low levels of hedging activity and, a period when the MBA Refi-index was high and volatile, implying greater hedging pressures. For this purpose we split the sample period into two sub-samples, 1997:01 – 2000:12 and 2001:01 – 2003:05, and estimate the model for these two sub-periods. During the first period, the mean value of the weekly Refi-index was 837 with a standard deviation of 651, while during the second period, the mean value was 3467 with a standard deviation of 2076. The results are in Exhibit 7.

Here we find a result that is counter-intuitive. During a period when the refi-incentives were low and when we might have expected to find that the amplification parameter β to be low in magnitude, we instead find it to be high not only for the 3mx10y swaption structure, but for all of the others as well. Furthermore, the estimates all have high p-values and t-statistics. Equally counter-intuitive, for the period 2001:01 – 2003:05, when the MBA refi-index was high and hedging pressures were great, we find that the values of the amplification parameter β implied that higher levels of hedging actually lowered volatility. Furthermore these results are highly statistically significant. Were the original Perli and Sack specification correct, we would be unlikely to observe this magnitude of parameter instability.

⁷ We identified a problem with the specification of the Likelihood Function in the original Perli and Sack analysis, and corrected it for the estimates provided in Exhibit 6. The corrected Likelihood estimates did not materially differ. A derivation of the corrected Likelihood function can be found in Appendix 1.

To further refine our understanding of the parameter instability in the Perli and Sack model, we examine the parameter estimates of a rolling window. Specifically we take 123-week intervals starting with the first week of 1997 and estimate the model parameters for each data window. If the model were correctly specified, these estimates would be constant, except for sampling error. Alternatively, if the stability were caused by an outlier in the data, we would see substantial movement in the parameter estimates as the outlier data points move into or out of the sample window.

Exhibit 8 below plots the amplification parameter estimate β as a function of the start date of the 123-week window. This graph exhibits two discontinuity points, one at about 3/31/1997 and the other at about 9/30/1998. These two points correspond to instances in which market disruptions brought about by LTCM enter and exit, respectively, from the sample window. Further, removing the few data observations associated with LTCM market disruption and re-estimating, results in dramatically different coefficient estimates that, for example, suggest that hedging adds to market stability in the pre-LTCM periods.

Conclusion

Understanding volatility and its determinants is an important open issue. The possibility that risk mitigation strategies could have the unintended consequence of increasing the amplification of exogenous shocks to financial markets raises important policy questions. Perli and Sack ask an important question relating to mortgage risk management implications for interest rate volatility.

Their analysis focused on explaining the component of short-term volatility unrelated to movements in long-term volatility. While their empirical findings that linked mortgage hedging to increased volatility are an artifact of several outlier observations in the data, modifying their analysis by dropping periods associated with the LTCM crisis and 9/11, generate estimates that suggest there is little impact to interest rate volatility from mortgage hedging. Indeed, over some periods the proxies for hedging activity appear to mitigate some of the market volatility.

While this analysis represents a step towards better understanding the linkage between hedging activities and financial stability, we believe that a more structural approach to this problem is needed. The term structure of volatility, for example, would suggest an alternative empirical approach to obtaining information about short-term volatility unrelated to long-term volatility movements than was taken in the Perli and Sack first stage regression. Further, the component of short-term volatility not explained by movement in long-term volatility is small in

both absolute and relative magnitude. Any theory that has policy relevance needs to explain material movements in market volatility and ideally, across the entire volatility surface. In addition, if market dynamics are different during periods of stress, as suggested by Cohen and Shin [2003], a modeling strategy capturing the differential impacts of hedging may be warranted.

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Exhibit 1: Definitions and Descriptive Statistics for Variables used in the Analysis: Jan. 1997– May 2003⁸

Variable	Definition	Source	Frequency	Mean	Standard Deviation
Mortgage Market Convexity	Duration exposure to be covered as a result of a rate shock at any point in time	Data used in the Perli and Sack paper	Weekly	-0.475	0.231
3mx10y Implied Volatility	Annual basis points (bps.) implied volatility.	Data used in the Perli and Sack paper	Weekly	1.164	0.445
2yx10y Implied Volatility	Annual basis points (bps.) implied volatility.	Data used in the Perli and Sack paper	Weekly	0.983	0.190
3mx10y Implied Volatility	Annual basis points (bps.) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.199	0.478
6mx10y Implied Volatility	Annual basis points (bps.) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.204	0.443
2yx10y Implied Volatility	Annual basis points (bps.) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.150	0.328
2yx5y Implied Volatility	Annual basis points (bps.) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.247	0.375
3mx5y Implied Volatility	Annual basis points (bps.) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.235	0.537
6mx5y Implied Volatility	Annual basis points (bps.) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.253	0.512

Note: The sample period begins with the week ending January 1997 and ends with the last week of May 2003, providing a total of 328 observations.

⁸ The Annual basis points (bps.) implied volatility is divided by 100. In doing so we are staying consistent with Perli and Sack (2003). Market convention, however, would express the implied volatility of a 3mx10y swaption as 116.4 basis points. This convention has no implications for the foregoing econometric exercise.

Exhibit 2: Time Series of the Perli and Sack and Our Values (Salomon Smith Barney) of Implied Volatility

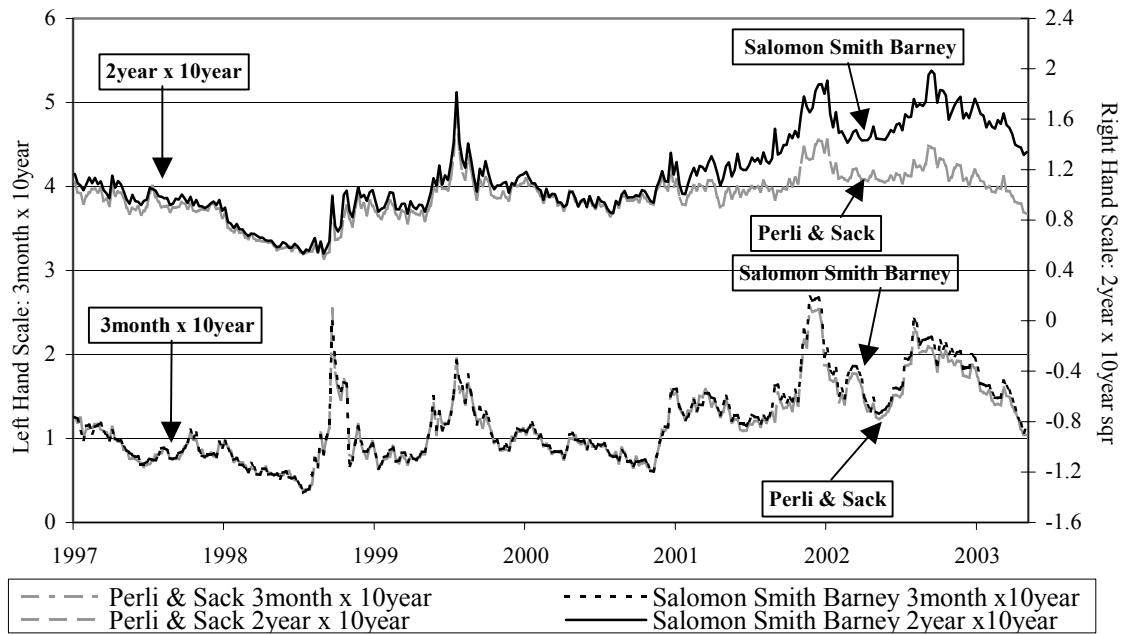


Exhibit 3: Regression Results for Short-Term on Long-Term Implied Variance

Dependent Variable	Independent Variable	Time Period Evaluated	R-Square	Estimated Parameters*	
				ψ_0	ψ_1
3mx10y	2yx10y	1997:01 – 2003:05	0.60	-0.631	1.827
3mx10y	2yx10y	1997:01 – 2003:05	0.7795	-0.2811	1.2876
6mx10y	2yx10y	1997:01 – 2003:05	0.8892	-0.2633	1.2765
3mx5y	2yx5y	1997:01 – 2003:05	0.8295	-0.3891	1.3022
6mx5y	2yx5y	1997:01 – 2003:05	0.9143	-0.3743	1.3051

*All parameters are significant at 99% level.

Exhibit 4: Time Series Comparison of Perli and Sack vs. our Dependent Variable

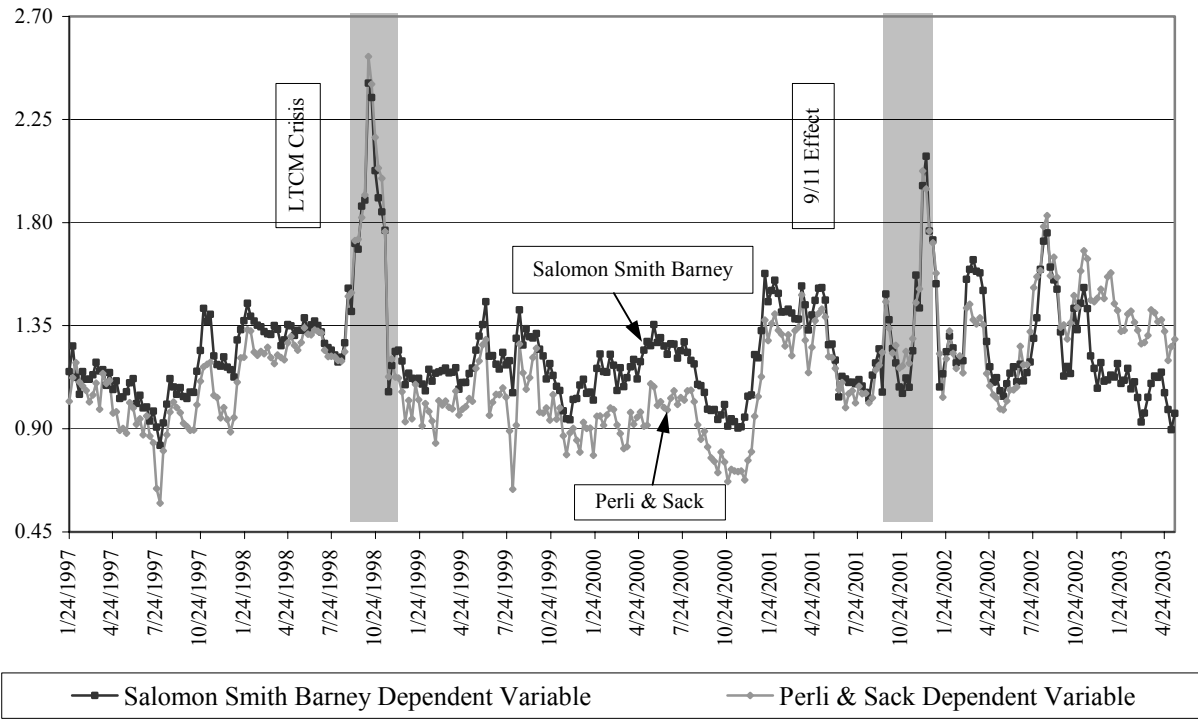


Exhibit 5: Regression Results for Orthogonalized Implied Variance

Dependent Variable	Time Period Evaluated	Mortgage Market Convexity As Independent Variable		
		Estimated Parameters (p-values in Parenthesis)		
		α_0	α_1	$\alpha_0\beta$
3mx10y	1997:01 – 2003:05	0.1000 (0.00)	0.8856 (0.00)	-0.0846 (0.00)
6mx10y	1997:01 – 2003:05	0.1511 (0.00)	0.8684 (0.00)	-0.0146 (0.41)
3mx5y	1997:01 – 2003:05	0.1395 (0.00)	0.8746 (0.00)	-0.0314 (0.22)
6mx5y	1997:01 – 2003:05	0.1646 (0.00)	0.8597 (0.00)	-0.0228 (0.22)

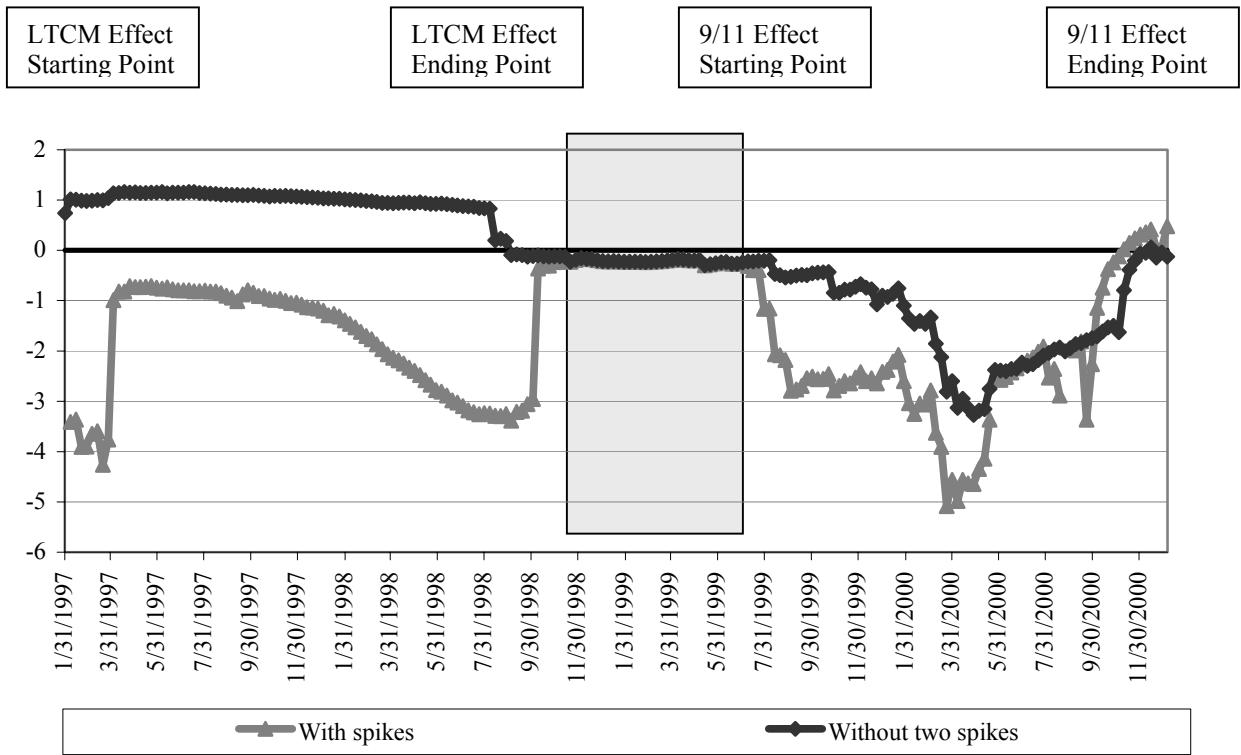
Exhibit 6: MLE Results for Orthogonalized Implied Variance

Dependent Variable	Time Period Evaluated	Mortgage Market Convexity As Independent Variable			
		Estimated Parameters (p-values in Parenthesis)			
		α_0	α_1	β	\mathcal{U}
3mx10y	1997:01 – 2003:05	0.0792 (0.00)	0.908 (0.00)	-0.834 (0.00)	0.0058 (0.00)
6mx10y	1997:01 – 2003:05	0.1093 (0.00)	0.892 (0.00)	-0.393 (0.01)	0.0037 (0.00)
3mx5y	1997:01 – 2003:05	0.0816 (0.00)	0.906 (0.00)	-0.917 (0.00)	0.0053 (0.00)
6mx5y	1997:01 – 2003:05	0.1273 (0.00)	0.879 (0.00)	-0.399 (0.00)	0.0039 (0.00)

Exhibit 7: MLE Results for Orthogonalized Implied Variance

Dependent Variable	Time Period Evaluated	Mortgage Market Convexity As Independent Variable			
		Estimated Parameters (p-values in Parenthesis)			
		α_0	α_1	β	ν
3mx10y	1997:01 – 2000:12	0.0529 (0.05)	0.930 (0.00)	-1.794 (0.00)	0.0038 (0.00)
	2001:01 – 2003:05	0.2958 (0.05)	0.838 (0.00)	0.476 (0.00)	0.0294 (0.00)
6mx10y	1997:01 – 2000:12	0.0671 (0.01)	0.928 (0.00)	-0.878 (0.01)	0.0025 (0.00)
	2001:01 – 2003:05	0.3319 (0.01)	0.784 (0.00)	0.292 (0.02)	0.0107 (0.00)
3mx5y	1997:01 – 2000:12	0.0707 (0.01)	0.917 (0.00)	-1.334 (0.00)	0.0039 (0.00)
	2001:01 – 2003:05	0.2966 (0.04)	0.870 (0.00)	0.620 (0.00)	0.0467 (0.00)
6mx5y	1997:01 – 2000:12	0.0926 (0.00)	0.907 (0.00)	-0.714 (0.00)	0.0027 (0.00)
	2001:01 – 2003:05	0.3574 (0.00)	0.793 (0.00)	0.364 (0.01)	0.0135 (0.00)

Exhibit 8: Changing MLE Estimates of Sensitivity Parameter



Appendix: Likelihood Function

For a variable x that follows a normal distribution $N(\mu_x, \sigma_x^2)$, with mean μ_x and variance σ_x^2 , the likelihood function, or the density function, is as follows:

$$L = f(x) = \frac{1}{(2\pi)^{\frac{1}{2}} \sigma_x} \exp\left(-\frac{(x - \mu_x)^2}{2\sigma_x^2}\right).$$

The log-likelihood function is the result of taking the natural log on both sides of the above equation: $\log L = -\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln \sigma_x^2 - \frac{1}{2\sigma_x^2} (x - \mu_x)^2$.

$$\text{To maximize } \log L \text{ is to maximize } -\frac{1}{2} \ln \sigma_x^2 - \frac{1}{2\sigma_x^2} (x - \mu_x)^2.$$

From equation (7) we have: $\sigma_{\Delta r, t}^2 = \alpha_0 + \alpha_0 \beta X_{t-1} + \alpha_1 \sigma_{\Delta r, t-1}^2 + (1 + \beta X_{t-1}) u_t$.

We can use ε_t to denote the amplified error term, with $\varepsilon_t = (1 + \beta X_{t-1}) u_t$. ε_t follows a normal distribution $N(0, V(1 + \beta X_{t-1})^2)$.

Our x variable is $\sigma_{\Delta r, t}^2$, with $\sigma_x^2 = V(1 + \beta X_{t-1})^2$ and $\mu_x = \alpha_0(1 + \beta X_{t-1}) + \alpha_1 \sigma_{\Delta r, t-1}^2$.

Substituting these terms into the log-likelihood function, we have:

$$L(\alpha_0, \alpha_1, \beta, \nu) = -\frac{1}{2} \ln(\nu(1 + \beta X_{t-1})^2) - \frac{1}{2\nu(1 + \beta X_{t-1})^2} (\sigma_{\Delta r, t}^2 - \alpha_0(1 + \beta X_{t-1}) - \alpha_1 \sigma_{\Delta r, t-1}^2)^2,$$

instead of

$$L(\alpha_0, \alpha_1, \beta, \nu) = -\frac{1}{2} \ln(\nu(1 + \beta X_{t-1})^2) - \frac{1}{2\nu} (\sigma_{\Delta r, t}^2 - \alpha_0(1 + \beta X_{t-1}) - \alpha_1 \sigma_{\Delta r, t-1}^2)^2 \text{ as in equation (8).}$$